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Research Article

Block Second Derivative Methods for the Direct Solution of Second Order Initial Value Problems of (ODEs) Using Lucas Polynomials as Basis Function

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Abstract
This paper presents a self - starting block method for the direct solution of general second order initial value problems of ordinary differential equations. The method was developed via interpolation and collocation of the Lucas polynomial as basis function. A continuous linear multistep method was generated and was evaluated at some desired points to give the discrete block method. The block method was investigated and was found to be consistent, zero stable and convergent. The method was applied on some second order initial value problems of ordinary differential equations and the performance was relatively better than those constructed by Awari et al and Jator et al respectively.

Keywords: *collocation, interpolation, Lucas polynomials, block method, discrete method, consistent, zero stable, Convergent.*

1. Introduction

Ordinary Differential Equations (ODEs) play a fundamental role in mathematical modeling across various scientific and engineering disciplines. They describe how a function evolves over time based on its derivatives, making them essential tools for analyzing dynamic systems. In this article, we propose the development of Second Derivative Block Linear Multistep (SEDEBLIM) methods for the direct solution of second-order ordinary differential equations of the form:

$$y''=f\left(x,y,y'\right),\quad y\left(x_0\right)=y_0\ ,\ y'\left(x_0\right)=y'_0\quad ,\quad x\in[a,b] \tag{1}$$

where f satisfies the Lipschitz condition.

There are currently two well - known techniques for solving (1). The first is to reduce (1) to a system of first order ordinary differential equation and then solve using predictor corrector or Runge-Kutta method. The second approach is to solve (1) directly using the block method since it preserves the traditional advantage of one step methods, of being self-starting and permitting easy Change of step length [7]. Its advantage over Runge-Kutta methods lies in the fact that they are less expensive in terms of the number of functions evaluation for a given order [6] suggests that this equation, along with its associated conditions, can be conventionally solved by reducing it to a first-order equivalent, which can then be addressed using any suitable numerical method. Traditional numerical techniques like (Euler's Method, Classical Runge-Kutta Methods, Adams-Bashforth Methods, Backward Differentiation Formulas (BDF), Finite Difference Methods) often convert second order differential equations into systems of first-order ODEs before applying standard solvers. However, this transformation increases computational complexity, memory consumption, and function evaluations, making it less efficient for large-scale problems [7]. Second derivative methods improve accuracy and stability by incorporating second derivative information in solving ODEs.

Unlike conventional single-derivative techniques, these methods utilize both first and second derivatives, which reduce truncation errors and enhances solution precision [5]. They are particularly effective for stiff ODE problems, where traditional solvers such as Euler's method and classical Runge-Kutta methods struggle with stability and demand very small step sizes [12], [5] further advanced second derivative methods by introducing a higher-order multistep scheme designed specifically for stiff equations, improving both efficiency and stability in challenging computational environments. To address these challenges, researchers have developed Second Derivative Block Linear Multistep (SEDEBLIM) Methods, which solve second-order ODEs directly without converting them into first-order systems.

The improved Second-Derivative Block Linear Multi-Step (SEDEBLIM) methods have proven transformative across multiple disciplines by efficiently solving complex second-order ODEs [17]. In physics, they enable high-fidelity simulations of celestial mechanics, including spacecraft trajectory optimization and n-body gravitational systems, where their simple structure preserves energy in long-term orbital integrations [15]. The methods also excel in quantum dynamics and electromagnetic wave propagation, providing stable solutions to time-dependent Schrödinger equations and Maxwell's equations in dispersive media. Engineering applications leverage SEDEBLIM's stability for critical vibration analysis [4], from aircraft flutter prediction to seismic response modeling, while robotics and autonomous systems employ them for real-time trajectory planning of mechanical systems [13]. In applied mathematics, these methods advance biomechanical modeling of muscle-tendon dynamics and neuron activation patterns, while adapted versions handle stochastic financial models.

The parallel architecture of SEDEBLIM methods particularly benefits large-scale problems, though challenges remain in memory-efficient implementations for embedded systems and initialization of irregular domains. These diverse applications demonstrate how SEDEBLIM methods bridge theoretical numerical analysis with cutting-edge computational demands across scientific and industrial domains.

However, [3] argues that these conventional methods like the Adams and Runge- kutta methos fail to fully exploit all the available information inherent in certain ordinary differential equations, particularly the oscillatory nature of their solutions. Furthermore, as pointed out by [1] and [17], reducing these equations to first-order systems not only increases their dimensionality but also adds substantial computational load, potentially compromising both accuracy and time efficiency. [15]

2. Derivation of the method

Consider an approximate solution of (1) presented by the Lucas polynomials of degree m+t+s-1 of the form;

$$y(x)=\sum_{i=0}^{m+t+s-1} c_i z_i(x) \ , \tag{2}$$

Where $z_i \in \mathbb{R}, y \in C^2(a,b)$,

The second derivative of (2) gives

$$y''(x)=\sum_{i=0}^{m+t+s-1} c_i z_i''(x)=f\left(x,y\left(x\right),y'\left(x\right)\right) \tag{3}$$

m is the interpolation point, while t and s are the collocation points of the first and second derivatives respectively. Inspired by the idea of [5], we Interpolate (2) at x_{n+k-1} and collocate its first and second derivatives at $x_{n+v}, v=0\left(1\right)k$ and x_{n+v} respectively to obtain a nonlinear equation of the form;

$$Ax=b$$

Where,

$$A=\begin{bmatrix} z_0(k-1) & z_1(k-1) & z_2(k-1) & z_3(k-1) & z_4(k-1) & \cdots & z_m(k-1) \\ z_0'(0) & z_1'(0) & z_2'(0) & z_3'(0) & z_4'(0) & \cdots & z_m'(0) \\ z_0'(h) & z_1'(h) & z_2'(h) & z_3'(h) & z_4'(h) & \cdots & z_m'(h) \\ z_0'(2h) & z_1'(2h) & z_2'(2h) & z_3'(2h) & z_4'(2h) & \cdots & z_m'(2h) \\ . & . & . & . & . & \cdots & . \\ . & . & . & . & . & \cdots & . \\ z_0'(h(k-1)) & z_1'(h(k-1)) & z_2'(h(k-1)) & z_3'(h(k-1)) & z_4'(h(k-1)) & \cdots & z_m'(h(k-1)) \\ z_0'(kh) & z_1'(kh) & z_2'(kh) & z_3'(kh) & z_4'(kh) & \cdots & z_m'(kh) \\ z_0''(kh) & z_1''(kh) & z_2''(kh) & z_3''(kh) & z_4''(kh) & \cdots & z_m''(kh) \end{bmatrix}, x=\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ . \\ . \\ . \\ c_m \end{bmatrix}, b=\begin{bmatrix} y_{n+k-1} \\ f_n \\ f_{n+1} \\ f_{n+2} \\ . \\ . \\ f_{n+k-1} \\ f_{n+k} \\ g_{n+k} \end{bmatrix}$$

Solving (2) for $c_i's, i=0(1)\,\mathrm{m+t+s-1}$ using the matrix inversion technique and substituting the values of the $c_i's$ obtained into (5) produces the continuous form of the Second Derivative Block Linear Multistep method of the form;

$$y(\mathrm{x})=\alpha_{k-1}(\mathrm{x})\,\mathrm{y}_{n+k-1}+h\sum_{j=0}^{k}\beta_j(\mathrm{x})\,\mathrm{f}(\mathrm{x}_{n+j},\mathrm{y}_{n+j})+h^2\gamma_k(\mathrm{x})g_{n+k} \qquad (4)$$

Where $\alpha_j(\mathrm{x}),\beta_j(\mathrm{x})$ and $\gamma_j(\mathrm{x})$ are the continuous coefficients of the method.

Evaluating (4) at $x_{n+v}, \mathrm{v}=0(1)\mathrm{k}$, except $k-1$, we obtain k discrete schemes. Putting together these discrete schemes obtained from these evaluations yields $2k$ discrete block discrete schemes needed for the computations.

3. Specification of the method

Considering k = 4, the interpolation of (2) at the points x_{n+k-1}, and collocating (4) at the points $x_n, x_{n+1}, x_{n+2}, x_{n+3}$ *and* x_{n+4} for the first derivative and collocating at x_{n+k} the second derivative, solving for the $c_i's$ and substituting in (2), leads to the continuous linear multistep method of the form

$$y(\mathrm{x})=\alpha_3(\mathrm{x})\,\mathrm{y}_{n+3}+h\sum_{j=0}^{4}\beta_j(\mathrm{x})\,\mathrm{f}(\mathrm{x}_{n+j},\mathrm{y}_{n+j})+h^2\gamma_4(\mathrm{x})g_{n+4} \qquad (5)$$

Where,

$$\begin{bmatrix} \alpha_0=1 \\ \beta_0=x-\frac{18327}{49280}h-\frac{793}{924h}x^2+\frac{285}{1232h^2}x^3+\frac{125}{4224h^3}x^4-\frac{137}{6160h^4}x^5+\frac{113}{44352h^5}x^6 \\ \beta_1=-\frac{459}{385}h+\frac{120}{77h}x^2-\frac{64}{77h^2}x^3+\frac{7}{132h^3}x^4+\frac{31}{770h^4}x^5-\frac{17}{2772h^5}x^6 \\ \beta_2=-\frac{1053}{1120}h-\frac{9}{7h}x^2+\frac{9}{7h^2}x^3-\frac{11}{32h^3}x^4+\frac{1}{70h^4}x^5+\frac{1}{336h^5}x^6 \\ \beta_3=-\frac{453}{770}h+\frac{232}{231h}x-\frac{96}{77h^2}x^2+\frac{71}{132h^3}x^4-\frac{69}{770h^4}x^5+\frac{13}{2772h^5}x^6 \\ \beta_4=\frac{4563}{49280}h-\frac{129}{308h}x+\frac{691}{1232h^2}x^3-\frac{1169}{4224h}x^4+\frac{353}{6160h^4}x^5-\frac{181}{44352h^5}x^6 \\ \gamma_4=-\frac{81}{2464}h+\frac{27}{154}x^2-\frac{75}{308h}x^3+\frac{75}{352h^2}x^4-\frac{9}{308h^3}x^5+\frac{3}{1232h^4}x^6 \end{bmatrix} \qquad (6)$$

Evaluating (6) at $x = 0,1,2,\ and\ 4$ with $h = x_{n+1} - x_n$ and the results substituted in (5), the following discrete schemes were obtained

$$y_{n+1} = \frac{1}{90}hf_n + y_{n+3} - \frac{17}{45}hf_{n+1} - \frac{19}{15}hf_{n+2} + \frac{1}{90}hf_{n+3} + \frac{17}{45}hf_{n+4}$$

$$y_{n+2} = y_{n+3} - \frac{11}{288}h^2 g_{n+4} - \frac{11}{1920}hf_n + \frac{7}{35}hf_{n+1} + \frac{83}{160}hf_{n+2} - \frac{19}{30}hf_{n+3} + \frac{1831}{17280}hf_{n+4}$$

$$y_{n+3} = y_n - \frac{3}{32}h^2 g_{n+4} + \frac{201}{640}hf_n + \frac{7}{5}hf_{n+1} + \frac{99}{160}hf_{n+2} + \frac{9}{10}hf_{n+3} - \frac{149}{640}hf_{n+4}$$

$$y_{n+4} = y_{n+3} - \frac{3}{32}h^2 g_{n+4} - \frac{17}{5760}hf_n + \frac{1}{45}hf_{n+1} - \frac{41}{480}hf_{n+2} + \frac{47}{90}hf_{n+3} + \frac{3133}{5760}hf_{n+4}$$

$$g_n = \frac{16}{3h}f_{n+1} - \frac{6}{h}f_{n+2} + \frac{16}{3h}f_{n+3} - \frac{7}{3h}f_{n+4} - g_{n+4} - \frac{7}{3h}f_n$$

$$g_{n+1} = \frac{9}{4h}f_{n+2} - \frac{7}{6h}f_{n+1} - \frac{3}{2h}f_{n+3} + \frac{29}{48}f_{n+4} - \frac{1}{4}g_{n+4} - \frac{3}{16h}f_n$$

$$g_{n+2} = \frac{4}{3h}f_{n+3} - \frac{1}{2h}f_{n+2} - \frac{4}{9h}f_{n+1} - \frac{31}{72}f_{n+4} + \frac{1}{6}g_{n+4} + \frac{1}{24}f_n$$

$$g_{n+3} = \frac{1}{6}hf_{n+1} - \frac{3}{4h}f_{n+2} - \frac{1}{6h}f_{n+3} + \frac{37}{48h}f_{n+4} - \frac{1}{4}g_{n+4} - \frac{1}{48}hf_n \qquad (7)$$

The new block method is of order $(5,5,5,5,5,5,5,5)^T$ and error constant of

$$\left(-\frac{1}{756}, \frac{19}{10080}, -\frac{11}{1120}, \frac{41}{30240}, -\frac{2}{5}, \frac{6}{5}, -\frac{4}{5}, \frac{2}{5}\right)^T.$$

Consistency

Following [7] and [12] the block method is consistent since it has order $p \geq 1$.

Zero Stability

The schemes from (7) are said to be zero stable if the roots λ_r, $r = 1,...,n$ of the first characteristic polynomial $\breve{\rho}(z)$, defined by

$$\rho(\lambda) = \left|\lambda * A^{(1)} - A^{(0)}\right| \qquad (8)$$

Satisfies $|z_r| \leq 1$ and every root with $|z_r| = 1$ has multiplicity not exceeding power of the differential equation in the limit as h→0.

Zero stability of the SEDEBLIM method k = 4

The Schemes from method k = 4 in (7) expressed in block form and in the stability polynomial (8) with particular matrices indicated below as:

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ y_{n+4} \end{bmatrix}+\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} y_{n-3} \\ y_{n-2} \\ y_{n-1} \\ y_{n} \end{bmatrix}+h\begin{bmatrix} -\frac{1}{90} & \frac{17}{45} & \frac{19}{15} & \frac{17}{45} \\ -\frac{1831}{17280} & \frac{19}{30} & \frac{83}{160} & -\frac{7}{135} \\ \frac{49}{640} & -\frac{9}{10} & -\frac{99}{160} & \frac{7}{5} \\ -\frac{3133}{5760} & -\frac{47}{90} & \frac{41}{480} & -\frac{1}{45} \end{bmatrix}\begin{bmatrix} f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+4} \end{bmatrix}+h\begin{bmatrix} 0 & 0 & 0 & -\frac{1}{90} \\ 0 & 0 & 0 & \frac{11}{1920} \\ 0 & 0 & 0 & -\frac{201}{640} \\ 0 & 0 & 0 & \frac{17}{5760} \end{bmatrix}\begin{bmatrix} f_{n-3} \\ f_{n-2} \\ f_{n-1} \\ f_{n} \end{bmatrix}+h^2\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{11}{288} \\ 0 & 0 & 0 & \frac{-3}{32} \\ 0 & 0 & 0 & \frac{3}{32} \end{bmatrix}\begin{bmatrix} g_{n+1} \\ g_{n+2} \\ g_{n+3} \\ g_{n+4} \end{bmatrix} \quad (9)$$

$$+h^2\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} g_{n-3} \\ g_{n-2} \\ g_{n-1} \\ g_{n} \end{bmatrix}$$

where,

$A^{(1)} =$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}, A^{(0)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \mathrm{B}^{(1)} = \begin{bmatrix} -\frac{1}{90} & \frac{17}{45} & \frac{19}{15} & \frac{17}{45} \\ -\frac{1831}{17280} & \frac{19}{30} & \frac{83}{160} & -\frac{7}{135} \\ \frac{49}{640} & -\frac{9}{10} & -\frac{99}{160} & \frac{7}{5} \\ -\frac{3133}{5760} & -\frac{47}{90} & \frac{41}{480} & -\frac{1}{45} \end{bmatrix},$$

$$B^{(0)} = \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{90} \\ 0 & 0 & 0 & \frac{11}{1920} \\ 0 & 0 & 0 & -\frac{201}{640} \\ 0 & 0 & 0 & \frac{17}{5760} \end{bmatrix}, \mathrm{G}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{11}{288} \\ 0 & 0 & 0 & \frac{-3}{32} \\ 0 & 0 & 0 & \frac{3}{32} \end{bmatrix}, \mathrm{G}^{(0)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

Then as $h \to 0$, substituting $A^{(1)}$ and $A^{(0)}$ into (8) yields

$$\rho(\lambda) = \left| \lambda * \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right| = 0 \quad (11)$$

$$\rho(\lambda) = \begin{vmatrix} \lambda & 0 & -\lambda & 0 \\ 0 & 0 & -\lambda & \lambda \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & -\lambda & 0 \end{vmatrix} = 0 \quad (12)$$

solving for λ, gives: $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = 0$ and $\lambda_4 = 0$

From the block solution (8), we have $\lambda_r, r = 1,\dots,n$ this shows that our method is zero stable, since every root with $|z_r| = 1$ has multiplicity not exceeding power of the differential equation in the limit as h → 0.

Convergence

According to [5], [6] and [7], the block method is convergent since it is consistent and zero stable.

4. Numerical Experiments

In this section, we implement the proposed method to solve two second order initial value problems of ordinary differential equations and examine the efficiency and accuracy of the proposed block method. The absolute errors of the test problems in [1] and [12] which are both of order six are compared with the proposed method.

Example 1

Consider the IVP $y'' = 100y,\ y(0) = 1,\ y'(0) = -10,\ h = 0.01$. With exact solution $y(x) = e^{-10x}$. with the results shown in Table 1

Table 1. Results of problem 1, for *h* = 0.01

x	Exact Solution	Results of proposed method	Error in proposed method	Error in [1]
0.01	0.9048374180	0.90483741893184108670960969228	9.10×10^{-10}	1.350×10^{-7}
0.02	0.8187307531	0.818730755233739499551629180168	2.1×10^{-9}	3.660×10^{-7}
0.03	0.6703200460	0.740818224114721744742992770528	0.0704981781	6.050×10^{-7}
0.04	0.6703200460	0.670320050450994483620061066424	4.5×10^{-9}	8.500×10^{-7}
0.05	0.6065306597	0.60653066490708079456096127289	5.2×10^{-9}	1.100×10^{-8}
0.06	0.5488116361	0.548811642357536372744375035708	6.3×10^{-9}	1.370×10^{-8}
0.07	0.4965853038	0.496585311183850348227183893816	7.4×10^{-9}	1.450×10^{-8}
0.08	0.4493289641	0.449328972491874070221893023437	8.4×10^{-9}	1.600×10^{-8}
0.09	0.4065696597	0.40656966901540922740490911720	9.3×10^{-9}	1.760×10^{-8}
0.010	0.3678794412	0.367879451598760000329151955516	1.04×10^{-8}	1.950×10^{-8}

Example 2

Consider the IVP $y'' = y',\ y(0) = 0,\ y'(0) = -1,\ h = 0.1$ With exact solution as $y''(x) = 1 - \exp(x)$. with the results shown in Table 2.

Table 2. Results of problem 2, for *h* = 0.1

x	Exact Solution	Results of proposed method	Error in proposed method	Error in [1]
0.1	-0.105170918	-0.105170916732651445270728063802	1.3×10^{-9}	3.35×10^{-9}
0.2	-0.221402758	-0.221402754762468402134369498436	3.2×10^{-9}	3.30×10^{-8}
0.3	-0.349858808	-0.349858801927299739598213921294	6.10×10^{-9}	1.23×10^{-7}
0.4	-0.491824698	-0.491824690007591650816714692705	8.0×10^{-9}	3.16×10^{-7}
0.5	-0.648721271	-0.648721260260093626911678618847	1.07×10^{-8}	6.59×10^{-7}
0.6	-0.822118800	-0.822118785997937910869631184123	1.40×10^{-8}	1.21×10^{-6}
0.7	-1.013752707	-1.01375268873921299323928710937	1.8×10^{-8}	2.04×10^{-6}
0.8	-1.225540928	-1.22554090571624692425252491969	2.451081834	3.23×10^{-6}
0.9	-1.459603111	-1.45960308299644001044047903016	2.8×10^{-8}	4.88×10^{-6}
0.10	-1.718281828	-1.71828179307838295409220760139	3.5×10^{-8}	7.1×10^{-6}

5. Conclusion

The given result illustrates the desired behavior of a numerical solution, which is to resemble the theoretical solution of the issue. It has been demonstrated in this research that the method used in this study may also be used to create continuous collocation methods for solving Second order ordinary differential equations directly. This study presents a new block approach ($k = 4$) that is stable and convergent. According to the findings of the instances given, our approach outperformed those in [2], respectively.

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