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Music and Numbers

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Abstract

The carnatic music of south India is broadly classified into (tala) rhythmic and melodic (raga) structures. Aesthetics and complexity of Carnatic music is mainly based on Numbers especially through concepts like tala (rhythm) and raga (melody). Both mathematically and conceptually, Music and numbers are deeply intertwined. In melodic the swaras, ragas, sthayis (octaves), music composition structures, Frequency, over tones, Alapana and Kalpoana Swaras in Creative music are framed with numbers which are highly appreciable. So, 72 Major (Melakarta) Ragas and the system organizes the ragas into 72 parent scales, which are created by systematically varying the positions of the swaras. This is a perfect example of the mathematical arrangement of notes. The complex mathematical structures of compositions with rhythmic and melodic patterns exhibit symmetry and balance, reflecting their understanding of both music and numbers. The tala system is more complicated one which are mainly expressed through the percussion instruments. 7 talas, 35 talas, 175 talas, taladasa pranas, with jathi, yathi, gati, speed, prasthara are shows the number oriented tala system. Yati refers to the shape of rhythmic patterns based on numbers. Musical intervals often have simple mathematical ratios. In a tani avartanam, percussionists especially mridangam players engage in highly mathematical improvisations. They break down the tala cycle into intricate subdivisions and create rhythmic phrases that adhere to strict numerical patterns. In jugalbandi all artists must align their improvisations with the tala structure, often using numerical calculations to ensure that their rhythmic cycles synchronize.

Keywords: *Carnatic music, Melody, Rhythm, Tala, Tani Avarthanam, Jugalbandi.*

Numbers and Carnatic music

The traditional classical music of South India is Carnatic music, which is deeply rooted in mathematical principles. The carnatic music of south India is broadly classified into (tala) rhythmic and melodic (raga) structures. Aesthetics and complexity of Carnatic music is mainly based on Numbers especially through concepts like tala (rhythm) and raga (melody). Numbers play a major and significant role in shaping Carnatic music. The musical sound can be explained mathematically through acoustics which reveals extraordinary properties of numbers. Musical form is a plan (which will have perfect structures fixed to numbers) through which a short piece of music can be enlarged. When the number is extended through a mathematical formula the music piece is enlarged. Both mathematically and conceptually, Music and numbers are deeply intertwined.

1. **Numbers** in Raga (Melody)

1.1. Swara (Notes):

The fundamental of Carnatic music uses basic swaras (S, R, G, M, P, D, N) – 7.
The fundamental of Western music uses basic swaras (Do, Re, Mi, Fa, So, La, Ti) – 7.

These seven notes combine to form different *ragas*, with varying ascending (Arohana) and descending (Avarohana) scales.

Arohana of Major Raga (the ascending order) - 7

Avarohana of Major Raga (the decending order) - 7
These swaras further expand to form Swarasthanas - 12
These swarasthanas through vivadhi swaras further expand to form - 16

1.2. Ragas:

These above swaras further by permutation and combination form major ragas -72 (36 Suddha madhyama ragas and 36 Prathi madyama ragas which are seen in the form of 12 chakras with each chakra consists of 6 ragas) So 72 Major (Melakarta) Ragas and the system organizes the ragas into 72 parent scales, which are created by systematically varying the positions of the swaras. This is a perfect example of the mathematical arrangement of notes.

Janya Ragas: These 72 major ragas will give rise to many janya raga (new ragas) which will be infinity in numbers.

1.3. Structure in Compositions

Many Carnatic compositions with the divisions like pallavi, anupallavi and charanam are structured in mathematically precise ways, often involving intricate numerical divisions of beats, which are explored through rhythmic and melodic improvisation. They are constructed with different starting points (eduppus) and with many pauses or silent gaps with perfect number counts. Gamakas, the ornamentations of notes in Carnatic music, are often applied in specific patterns. While they add emotional depth to the music, the execution of gamakas often follows strict mathematical patterns, especially in improvisational sections. Many composers, like Muthu swami Dikshitar, *Shyama Shastri*, *Tyagaraja*, Thirugnana sambandar, Arunagirinadar,Papnasam Sivan embedded complex mathematical structures into their compositions, using rhythmic and melodic patterns that exhibit symmetry and balance, reflecting their understanding of both music and numbers.

1.4. Frequency and Octave

Frequency:

Frequency is the number of times a complete cycle of vibration occurs per second. It is measured in Hertz (Hz). In music the speed of vibration of a sound wave measured in cycles per second. It determines the pitch. The faster than frequency, the higher than pitch the frequencies 440 Hz and 880 Hz both corresponds to the mutual note A. But one octave apart. A frequency of a note is the number of vibrations (sound waves) per second, measured in Hertz (Hz). For example, the note has a frequency of 440 Hz. The relationship between different notes is often based on numerical ratios.

Calculating Frequency: F = 1/T

f- Frequency in Hertz. T – The time to complete one cycle in seconds.

Octaves

In mu8sic an octave is the interval between 2 pitches where one has a frequency that is twice as fast as the other. An octave is musical interval of eight notes. For Example – major scale of Western Music, the eight notes from low to high CDEFGAB, in Carnatic music SRGMPDNS .Sing from Do to Do on keys in western music and Sa to Sa in Carnatic music. Moving up an octave means doubling the frequency. If a first note of G sruthi is 440 Hz, then the first note of A sruthi (next sruthi) would have a frequency of 880 Hz (440 x 2). A human voice can sing easily 2 octaves while singing. The instruments can show many octaves according to the size and its construction. The next Pitch is called Octave because its eighth note (just as an octopus has eight legs). Piano has eight full octaves. In carnatic music it is called sthayi. There are five types of sthayis.

1. Madhya sthayi - middle octave - 7 notes
2. Taara sthayi - higher octave - 7 notes
3. Manthra sthayi - lower octave - 7 notes
4. Aditaara shayi - further higher octave - 7 notes
5. Anu manthra sthayi - further lower octave - 7 notes

Each sthayi will have 7 swaras which are called pentatonic scale. So totally five sthayis shows the range of 35 swaras.

1.5. Harmonics and Overtones:

In a Carnatic music in the sruthi or drone instrument shows overtones. By plucking the lower octave manthra sthayi string we can observe or hear anthara gantharam for manthra sathjam. This is the overtone which exibit double the frequency of the basic frequency. A fundamental frequency produces overtones or harmonics, which are whole-number multiples of the fundamental frequency (e.g., 2x, 3x, 4x the base frequency). These overtones create the timbre or color of a sound.

1.6. Mathematical Patterns in Swara Kalpana

Kalpana Swara:

Kalpana swara is one of the major divisions of Creative Music (Manodarma sangeetham). As an improvisational technique, the artist uses numbers to create patterns of swaras (notes) over a given tala. For example, an artist might sing in swara patterns like 3, 4, 5, or 7 notes per beat, creating rhythmic complexity. For example ga,ri,sa,ni,da in the form of five, ga,ree,sa, ni, da in the form of six, ga,ri,ga,ri,sa,ni,da in the form of seven. Carnatic musicians often use mathematical precision to modulate improvisation and explore different rhythmic and melodic variations. For example, when singing in Tisram (3 subdivisions), Chatusram (4 subdivisions), or Misram (7 subdivisions), they maintain a strict adherence to numerical patterns.

2. Tala and Mathematical numbers:

Music has been broadly divided into two sections called melodies and rhythms. Ragas come under melodies and tala comes under Rhythms.

Talas: In Carnatic music, the rhythmic framework of any musical piece is governed by talas. Talas are mathematical structures expressed in numbers. Talas shows cycles of beats. Highly complex structure of a Tala is framed by specific number of beats which has sub divisions. There are primarily seven basic talas. They are called Saptha talas means seven. Each tala will have varied Akshars in numbers according to the jathi.

Saptha talas are

1. Duruva tala
2. Matya tala
3. Rupaga tala
4. Thriputa tala
5. Jampa tala
6. Ata tala
7. Eka tala

2.1. Common Talas and their Numerical Structure

Here are some common talas in Carnatic music and their beat structures:

Tala	Structure (Angas)	Number of Beats
Adi Tala	1 Laghu (Chatusra Jati) + 2 Drutams	4 + 2 + 2 = 8
Rupaka Tala	1 Drutam + 1 Laghu (Chatusra Jati)	2 + 4 = 6
Jhampa Tala	1 Laghu (Misra Jati) + 1 Anudrutam + 1 Drutam	7 + 1 + 2 = 10
Triputa Tala	1 Laghu (Tisra Jati) + 2 Drutams	3 + 2 + 2 = 7
Dhruva Tala	1 Laghu + 1 Drutam + 1 Laghu	4 + 2 + 4 = 10

2.2. Jathi (Subdivisions): Each tala can be subdivided into different jathis or beat patterns. For example, a beat in a cycle can be divided into

1. 3 (Tisra jathi - 3 counts
2. 4 (Chatusra jathi) – 4 counts
3. 5 (Khanda jathi - 5 counts
4. 7 (Misra jathi) - 7 counts
5. 9 (Sankeerna jathi) – 9 counts

2.3. Thirty five talas:

These jathis add talas to the mathematical intricacies of the rhythm. These saptha talas with this five jathis that is tisra jathi, chatusra jathi, kandha jathi, misra jathi, sankeerna jathi will create 35 talas - 7 X 5 = 35

2.4. Nadais (Subdivisions of Beat)

In Carnatic music, the division of the beat (*Nadais*) plays a crucial role in rhythm. The basic beat can be subdivided in different ways, there are five types of nadais otherwise called gatis. For example, in Adi Tala (8 beats), if the musician chooses Tisra gati, each beat is subdivided into 3, resulting in 8 x 3 = 24 sub-beats in one avartanam.

1.Tisra gati - (3 subdivisions per beat),
2.Chatusra gati - (4 subdivisions per beat)
3. kandha gati - (5 subdivisions per beat)

4. Misra gati - (7 subdivisions per beat)
5. Sankeerna gati - (9 subdivisions per beat)

35 TALAS – CHART

Suladhi Sapta Talas	Samskruta Name	Anga	Symbol	Technical Name	Samskruta Name	Symbol	No. of Beats
Dhruva Tala	Indraneelam	Laghu-Dhruta-Laghu-Laghu	IOII	Chaturasra Jati Dhruva Tala	Shrikara	$I_4 O I_4 I_4$	4 + 2 + 4 + 4 = 14
				Tisra Jati Dhruva Tala	Mani	$I_3 O I_3 I_3$	3 + 2 + 3 + 3 = 11
				Misra Jati Dhruva Tala	Poorana	$I_7 O I_7 I_7$	7 + 2 + 7 + 7 = 23
				Khanda Jati Dhruva Tala	Pramaana	$I_5 O I_5 I_5$	5 + 2 + 5 + 5 = 17
				Sankeerna Jati Dhruva Tala	Bhuvana	$I_9 O I_9 I_9$	9 + 2 + 9 + 9 = 29
Matya Tala	Mahavajram	Laghu-Dhruta-Laghu	IOI	Chaturasra Jati Matya Tala	Sama	$I_4 O I_4$	4 + 2 + 4 = 10
				Tisra Jati Matya Tala	Saara	$I_3 O I_3$	3 + 2 + 3 = 8
				Misra Jati Matya Tala	Udeema	$I_7 O I_7$	7 + 2 + 7 = 16
				Khanda Jati Matya Tala	Udaya	$I_5 O I_5$	5 + 2 + 5 = 12
				Sankeerna Jati Matya Tala	Raava	$I_9 O I_9$	9 + 2 + 9 = 20
Rupaka Tala	Nidrosham	Dhruta-Laghu	OI	Chaturasra Jati Rupaka Tala	Patti	$O I_4$	2 + 4 = 6
				Tisra Jati Rupaka Tala	Chakra	$O I_3$	2 + 3 = 5
				Misra Jati Rupaka Tala	Kula	$O I_7$	2 + 7 = 9
				Khanda Jati Rupaka Tala	Raaja	$O I_5$	2 + 5 = 7
				Sankeerna Jati Rupaka Tala	Bindu	$O I_9$	2 + 9 = 11
Jhampa Tala	Seeram	Laghu-Anudhruta-Dhruta	IUO	Chaturasra Jati Jhampa Tala	Madhura	$I_4 U O$	4 + 1 + 2 = 7
				Tisra Jati Jhampa Tala	Kadamba	$I_3 U O$	3 + 1 + 2 = 6
				Misra Jati Jhampa Tala	Sura	$I_7 U O$	7 + 1 + 2 = 10
				Khanda Jati Jhampa Tala	Chana	$I_5 U O$	5 + 1 + 2 = 8
				Sankeerna Jati Jhampa Tala	Kara	$I_9 U O$	9 + 1 + 2 = 12
Triputa Tala	Kokilam	Laghu-Dhruta-Dhruta	IOO	Chaturasra Jati Triputa Tala	Adi	$I_4 O O$	4 + 2 + 2 = 8
				Tisra Jati Triputa Tala	Shanka	$I_3 O O$	3 + 2 + 2 = 7
				Misra Jati Triputa Tala	Leela	$I_7 O O$	7 + 2 + 2 = 11
				Khanda Jati Triputa Tala	Dushkara	$I_5 O O$	5 + 2 + 2 = 9
				Sankeerna Jati Triputa Tala	Bhoga	$I_9 O O$	9 + 2 + 2 = 13
Ata Tala	Avartakam	Laghu-Laghu-Dhruta-Dhruta	IIOO	Chaturasra Jati Ata Tala	Lekha	$I_4 I_4 O O$	4 + 4 + 2 + 2 = 12
				Tisra Jati Ata Tala	Guptha	$I_3 I_3 O O$	3 + 3 + 2 + 2 = 10
				Misra Jati Ata Tala	Loya	$I_7 I_7 O O$	7 + 7 + 2 + 2 = 18
				Khanda Jati Ata Tala	Vidala	$I_5 I_5 O O$	5 + 5 + 2 + 2 = 14
				Sankeerna Jati Ata Tala	Dheera	$I_9 I_9 O O$	9 + 9 + 2 + 2 = 22
Eka Tala	Sadananda	Laghu	I	Chaturasra Jati Eka Tala	Maana	I_4	4
				Tisra Jati Eka Tala	Sudha	I_3	3
				Misra Jati Eka Tala	Raaga	I_7	7
				Khanda Jati Eka Tala	Raatha	I_5	5
				Sankeerna Jati Eka Tala	Vasu	I_9	9

2.5. 175 talas:

The above 35 talas with these 5 gatis that is is tisra gati, chatusra gati, kandha gati, misra gati, sankeerna gati will produce 175 talas - 35 X 5 = 175. These subdivisions, often rendered as numbers, help in improvisation and make complex rhythmic patterns possible.

2.7. Tala dasa Pranas:

There are ten tala dasa Pranas. They are kaalam, angam, kriyai, graham, jathi, layam, kalai, margam, Yathi, parastharam. They are the main ten factors that are governing the tala.

1.Kaalam – 3 First speed, Second speed, Third speed.
2. Angam – 6 Shadangam -6, Shodashangam – 16,
3. Kriyai – 2 Sasaptha kriyai, Nishabda kriyai,
4. Graham – 3 Sama graham, Atheetha graham, Anagatha graham
5. Jathi – 5 Tisra jathi, Chatusra jathi, Kandha jathi, Misra jathi, Sankeerna jathi
6. Layam – 3 vilamba layam, madyama layam, duritha layam
7. Kalai – 3 Oru kalai, Rendu kalai. Naangu kalai
8. Margam – 6 Shan margam
9. Yathi – 6 Srothavaga, Gopuchcha, Mridanga, Damaruga,Sama,Visama

2.7. Yati (Rhythmic Shapes)

Yati refers to the shape of rhythmic patterns based on numbers. There are six types of Yatis some common patterns in Yatis are:

1. **Srotovaha Yati** : The pattern expands (e.g., 3, 4, 5, 6 beats).
2. **Gopuccha Yati** : The pattern contracts (e.g., 6, 5, 4, 3 beats).
3. **Mridanga Yati** : The pattern is symmetrical (e.g., 3, 4, 5, 4, 3 beats).
4. **Damaruga Yati** : The pattern is symmetrical in reverse (5,4,3,4,5 beats)
5. **Sama Yati** : The pattern is equal – (2,2,2,2,2,2)
6. **Vishama Yati** : The pattern is unequal - (5,3,1,2,0,3,2)

These shapes provide visual and numerical beauty in rhythmic improvisation.

2. 8. Tani Avartanam (Percussion Solos).

Korvai and Mora (Mathematical Patterns)

In Carnatic percussion, particularly in all musical or Instrumental performances, The mridangam and other other percussion instruments like gatam, kanjira morsing,tabela will play thani Avartanam. The Tani Avartanam consists of complex rhythmic patterns which are built using numerical calculations. In a tani avartanam, percussionists especially mridangam players engage in highly mathematical improvisations. They break down the tala cycle into intricate subdivisions and create rhythmic phrases that adhere to strict numerical patterns.

Korvai:

A korvai is a rhythmic pattern that is repeated three times at the end of an improvisation or composition. The total number of beats in a korvai must fit perfectly into the tala cycle, so musicians calculate the number of beats in the pattern to ensure it aligns correctly. For example, a musician might design a korvai that fits exactly into a 16-beat cycle by breaking it into smaller subgroups (e.g., 5 + 5 + 6 = 16). A Korvai is a rhythmic pattern that is repeated three times and typically used to conclude a percussion solo. The calculation of the number of beats in a Korvai is critical for it to resolve at the appropriate point within the tala cycle.

Mora:

A Mora is another rhythmic pattern where a group of beats is repeated three times and concludes on the first beat of the tala. These patterns often follow numerical groupings like 3, 5, or 7, requiring mathematical calculation to fit the tala structure. So, Mora is another rhythmic pattern used in the Tani Avartanam, involving repetitive mathematical calculations to align the pattern with the tala.

Konnakol:

This is the vocal art of expressing rhythms, where musicians vocalize rhythmic syllables. It involves reciting rhythms in various subdivisions and repeating them in patterns based on numbers, often requiring complex mental calculations to ensure the rhythm resolves correctly at the end of the tala cycle.

2.9. Proportionality in Jugalbandi

Jugalbandi is a one where many and different Instruments of different music like Carnatic music, Western music, Hindustani music will join together to give performance. If it is a percussional jugalbandIi then the instruments played are mridangam, gatam,kanjira, morsing from Crnatic music, tabela from Hindustani music and drums from western music. In Carnatic jugalbandis numbers are used to maintain balance between the artists. All artists must align their improvisations with the tala structure, often using numerical calculations to ensure that their rhythmic cycles synchronize.

Carnatic music is not just a form of artistic expression but also a highly mathematical art form. The use of numbers and numerical patterns in rhythm, melody, and improvisation gives it both its complexity and beauty.

3. Time Signatures and numbers

Time signatures:

In Western music the meter of a piece of music is mentioned like 4/4 or ¾. This indicates the number of beats in a measure and the type of note gets one beat. In this 4/4 or ¾ the number on the top indicates the beats, and the number on the bottom indicates the type of note that represents one beat.

Division of time:

Rhythms are often based on dividing beats into halves1\2, quarter1\4, eighths 1\8, etc. The mathematical aspect of dividing and grouping these units is critical to creating the structure of music.

Scales: Scales in music are built on specific patterns of intervals. For example, the major scale follows a pattern of whole steps and half steps. This pattern translates into a numerical relationship between pitches. There many types of scale according to the number of notes They are hepta tonic, penta tonic, hexa tonic ad so on. The distance between two notes can be described numerically (e.g., a perfect fifth spans seven half steps). Musical intervals often have simple mathematical ratios.

4. Tuning Systems and numbers

Pythagorean Tuning: In Tamil it is called palai system. There are four types of palais. They are vatta palai, Chadurap paalai, Thirikona palai. In Vatta palai there are cycle of fifth and cycle of fourth. Through this numerous new ragas arises. These are calculated in mathematical fractions. Based on the mathematical relationships between intervals, particularly the ratio of 3:2 for a perfect fifth. The system is named after Pythagoras, who explored the connections between music and mathematics.

Equal Temperament: Modern Western music system uses the "equal temperament" tuning system, which divides the 12 notes in an octave into 12 equal parts. The frequency ratio between each note is the 12th root of 2 (≈1.05946).

5. Digital Music and Numbers

Sampling: Digital music relies on sampling, where sound waves are measured (sampled) thousands of times per second. This data is stored as numbers and can be manipulated with digital signal processing (DSP).

Bit depth and sample rate: Bit depth (e.g., 16-bit or 24-bit) and sample rate (e.g., 44.1 kHz or 96 kHz) in digital recording involve specific numerical values that affect the quality and resolution of sound.

Conclusion:

Carnatic music is not just a form of artistic expression but also a highly mathematical art form. The use of numbers and numerical patterns in rhythm, melody, and improvisation gives it both its complexity and beauty.

In Carnatic music, the tala system is a beautiful expression of how numbers and mathematics can shape rhythm, creating a complex yet cohesive structure for musicians to explore.

Music and numbers, though different in nature, are bound by a shared structure, creating a fascinating intersection between art and mathematics.

A fundamental frequency produces overtones or harmonics, which are whole-number multiples of the fundamental frequency (e.g., 2x, 3x, 4x the base frequency).

Bit depth (e.g., 16-bit or 24-bit) and sample rate (e.g., 44.1 kHz or 96 kHz) in digital recording involve specific numerical values

The complex mathematical structures of compositions with rhythmic and melodic patterns exhibit symmetry and balance, reflecting their understanding of both music and numbers.

The tala system is more complicated one which are mainly expressed through the percussion instruments. 7 talas, 35 talas, 175 talas, taladasa pranas, with jathi, yathi, gati,speed, prasthara are shows the number oriented tala system.

In a *tani avartanam*, percussionists break down the tala cycle into intricate subdivisions and create rhythmic phrases that adhere to strict numerical patterns.

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